COURSE DESCRIPTION CARD - SYLLABUS

Course name
Linear algebra [S1SI1E>ALIN]

Course

Field of study
Artificial Intelligence

Year/Semester
1/2

Area of study (specialization)
–

Profile of study
general academic

Level of study
first-cycle

Course offered in
English

Form of study
full-time

Requirements
compulsory

Number of hours

Lecture
30

Laboratory classes
0

Other
0

Tutorials
30

Projects/seminars
0

Number of credit points

5,00

Coordinators

prof. dr hab. inż. Adam Dąbrowski
adam.dabrowski@put.poznan.pl

Lecturers

prof. dr hab. inż. Adam Dąbrowski
adam.dabrowski@put.poznan.pl

dr hab. Szymon Drgas
szymon.drgas@put.poznan.pl

Prerequisites

Knowledge: The student starting this course should have knowledge of mathematics at level of secondary school. Skills: Should have the ability to solve basic problems of algebra and geometry and also the ability to gather information from indicated sources. Social competences: Should understand the necessity of widening his competences. Regarding social competences he/she should have such character traits like honesty, perseverance, curiosity, creativity, personal culture, respect for other people.

Course objective

none

Course-related learning outcomes

none

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

none

Programme content

The Linear Algebra subject covers linear algebra, analytic geometry, real and complex vector spaces and algebraic methods of describing and analyzing the properties of linear transformations and their applications to the analysis of the stability of dynamic systems, description of signal processing methods in the time and frequency domains, numerical simulation of dynamic systems in the continuous-time and discrete-time domains.

Course topics

Programme of the lecture includes:
1. Introduction to algebra and geometry (the notion of set, number sets, vectors, matrices, algebraic operations, modulo operations, operations on sets, quantifiers, Cartesian product, countable and uncountable sets, the notion of relation, binary relations, reflexive, symmetric, and transitive relations, ordering and semi-ordering relations, multi-value relations, the notion of function, injection, surjection, bijection, inverse function, multiplication of mappings, inner and outer operations, compatibility of relation and operation, algebraic structures, geometric illustrations of systems of linear equations: column and row interpretations.
2. Complex numbers (definition, canonical form, addition, subtraction, multiplication, division, conjugation, the Euler's equation, de Moivre equation, roots, powers, andogarithms of complex numbers, the applications of complex numbers in electrical engineering and electronics
3. The basic notions of the linear algebra (dot product, orthogonal projection of a vector onto vector, the equation of a line on a plane, the positive side of a line, the equation of a plane, the equation of a plane in 3D space, the equations of a line in 3D space, hyperplane in n-dimensional space, matrix by vector multiplication, row vector by matrix multiplication, the exchange of rows and columns of a matrix - permutation matrix, identity matrix, graphical picture of vectors, vectors in nature and engineering, basic operations on vectors, multiplication of matrices, inverse matrix, the determinant of a square matrix)
4. Linear space, column space and nullspace of a matrix, Gauss elimination, LU decomposition, the echelon form of a matrix, the notion of space and subspace, the sum and product of subspaces, determination of spaces and subspaces using homogeneous systems of equations, space spanned on vectors, linear dependence and independence, basis, dimensionality, column space, Gauss elimination - pivots, Gauss-Jordan elimination, the computation of inverse matrix using Gauss-Jordan method, LU decomposition, equivalent systems of equations, rank of matrix, solution to Ax=0, row-reduced echelon form.
5. Four fundamental spaces (basis of linear space, column space, row space, nullspace, left nullspace, relation between the spaces, additional information: linear transformation of linear spaces, the image and kernel of a linear transformation)
6. Systems of linear equations (matrix equation as the inverse problem, row interpretation, column interpretation, existence and uniqueness of the solution, augmented matrix, homogeneous system, the conditions of existence of the solutions of a system of linear equations, nonhomogeneous system of linear equations, solving of n equations with n unknowns, the solutions on Ax=b - case study, Kronecker-Capelli theorem, matrix determinant, Cramer's equations)
7. Formulating matrix equations in natural and engineering sciences (structural and flow graphs, Eulers theorem, topological proof to Euler's theorem, incidence matrix, the nullspace of incidence matrix, the second Kirchoff's law written with node potentials and an incidence matrix, admittance matrix, the example of analyzing simple electrical circuit, the method of closed-loop currents, the comparison of closed-loop currents and node potentials methods.
8. The change of a basis and the least squares method (coordinates and components of vectors, colour as a vector, linear transformations of linear spaces, the change of a basis, the matrix of transformation, change of a basis example, the conception of the projection of space to a subspace, projection onto subspace spanned on subset vectors of a basis, projection of a colour image, orthogonal projection, solving Ax=b which has not solutions (least squares method), linear regression)
9. Vectors, bases and orthogonal matrices, QR decomposition, dot product, orthogonal and orthonormal vectors, standard basis, orthogonal matrices examples, Hadamard matrix, orthogonal transformations, rotators and reflectors, QR decomposition and its ealization with rotators, orthogonal projection, Housholder reflector and its properties, the application of reflectors to QR decomposition, Gram-Schmidt orthogonalization)

10. Eigenvalues and eigenvectors (the great equation of algebra, eigenvalues and eigenvectors, geometric multiplicity, determining eigenvalues and eigenvectors, relation of determinant and trace to eigenvalues, the characteristic equation of a matrix, algebraic multiplicity of an eigenvalue, relation between algebraic and geometric multiplicities, simple and defective matrices, examples of linear transformations)
11. Eigenvalues of a matrix and stability of dynamic systems (eigshow example in Matlab environment, discrete dynamical system, diagonalization of a matrix, powers of matrix, asymptotic stability of a matrix, diagonalizable and non-diagonalizable matrices, the Fibonacci sequence, a system of linear differential equations, the general solutions of linear differential equations system, the stability of linear system of equations, similar matrices)
12. Positive definite matrices, Cholesky decomposition, similar matrices (eigenvalues and eigenvectors of symmetric matrices, diagonalization of symmetric matrices, Hermitian matrices, positive definite matrices, semi-definite and indefinite matrices, energetic definition of positive definite matrix. passive physical systems, quadratic form, construction of positive definite matrix, Cholesky decomposition, Jordan form, Jordan blocks, Jordan's theorem)
13. Singular value decomposition, pseudoinverse matrices (separation of acoustic signals using SVD, SVD of a positive definite matrix, web page search (HITS algorithm), left- and right- inverse matrices, Moore-Penrose pseudoinverse matrix)
14. Sensitivity of linear systems to measurement and arithmetic errors (measurement errors, arithmetic errors, sensitivity of systems of linear equations, norm of vector, norm of matrix, Frobenius norm, operator, p-power and spectral norm, column norm, system of linear equation with a disturbed vector of free elements, indicator of matrix conditioning, maximal and minimal lengthening of a vector by a matrix, ill-conditioned matrices, effects of the disturbance of a matrix)
15. Unitary matrices, DFT, Shur decomposition (dot product of vectors with complex components, unitary matrices, complex numbers in electrical engineering, waves in electrical circuit, incident, reflected, and transmitted wave, lossless electrical circuit, unitary transformation of signal, discrete Fourier transform, Schur decomposition, nilpotent matrices)
Program of tutorials includes topics from the lectures with emphasis on:
1. Algebraic structures
2. Complex numbers
3. Solving systems of linear equations using Gauss elimination
4. Matrix calculus, inverse matrix
5. LU decomposition, Cholesky decomposition
6. Linear spaces, the echelon form of a matrix, nullspace
7. Complete solution of a system of linear equations
8. Incidence matrix
9. Projection and least squares method
10. Orthogonality, Gram-Schmidt normalization, QR decomposition
11. Matrix determinant, Cramer's equations
12. Eigenvalues and eigenvectors, stability of dynamical systems
13. Positive definite matrices, similar matrices
14. SVD and pseudo inverse matrices
15. Sensitivity of linear systems

Teaching methods

1. Lecture: multimedia presentation, presentation supported with examples showed on the blackboard, solving problems, demonstration
2. Tutorials: solving problems, practical exercises, discussion

Bibliography

Dąbrowski A., "Algebra liniowa", zestaw sfilmowanych wykładów, www.put.poznan.pl, e- learning Moodle, wykłady otwarte, PolitechnikaPoznańska, Poznań 2020 oraz materiały do wykładów wraz z zadaniami egzaminacyjnymi z rozwiązaniami na stronie www.dsp.put.poznan.pl
2. G. Strang, http://ocw.mit.edu, wykłady z algebry liniowej Profesora Gilberta Stranga, Massachusetts Institute of Technology
3. G. Strang, Introduction to linear algebra, Wellesley-Cambridge Press, MA, 2009
4. T. Kaczorek, Wektory i macierze w automatyce i elektrotechnice, WNT, Warszawa 1998

Additional
1. D. S. Watkins, Fundamentals of matrix computations, John Wiley & Sons, New York, 1991
2. G. Strang, Computational Science and Engineering, Wellesley-Cambridge Press, MA, 2007
3. A. Jennings, Matrix computations for engineers and scientists, J. Wiley & Sons, New York 1977

Breakdown of average student's workload

	Hours	ECTS
Total workload	125	5,00
Classes requiring direct contact with the teacher	62	2,50
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	63	2,50